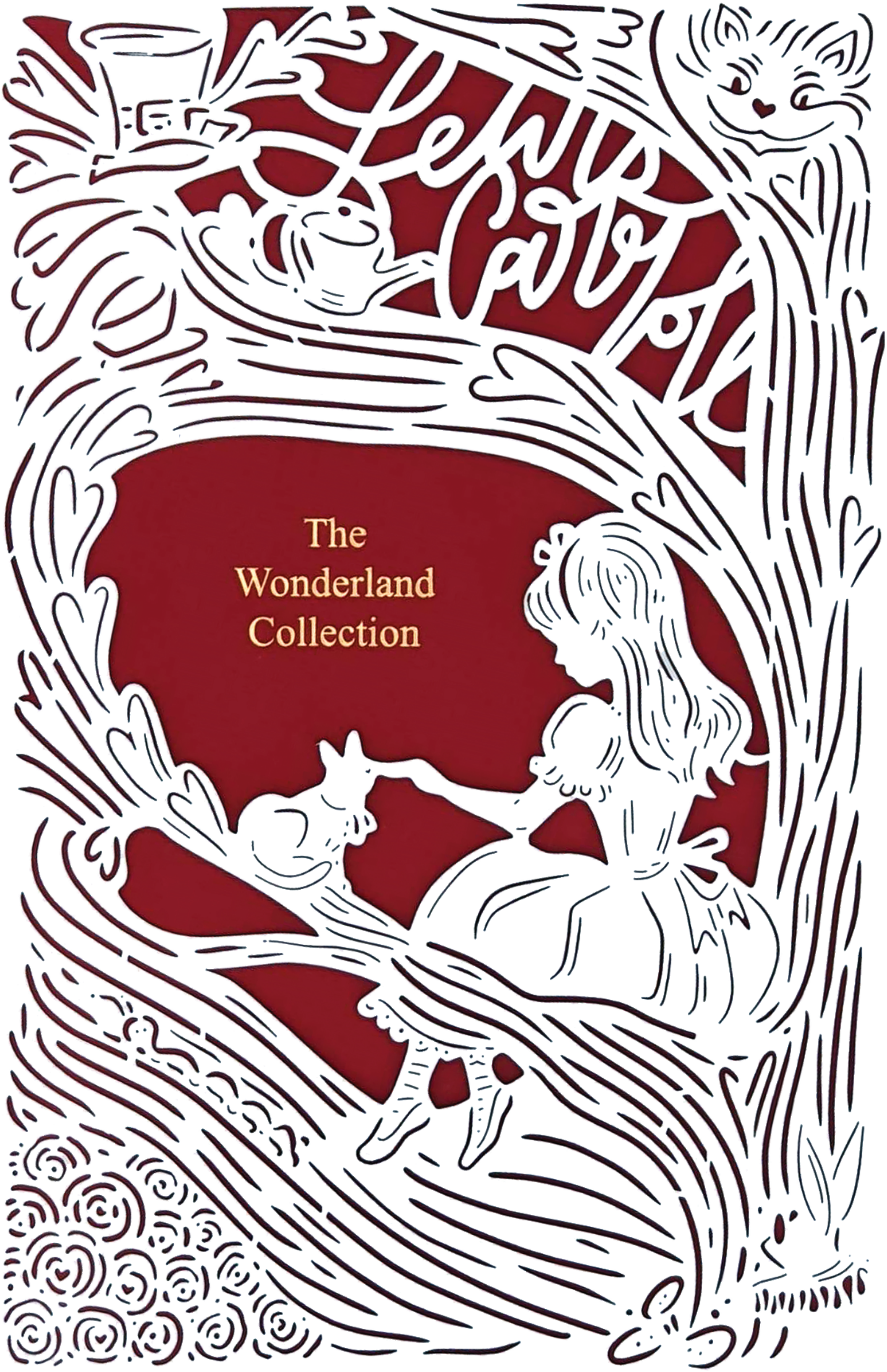


A PDF COMPANION TO THE AUDIOBOOK

APPENDIX

'A knot!' said Alice. 'Oh, do let me help to undo it!'

ANSWERS TO KNOT I.

Problem.—Two travellers spend from 3 o'clock till 9 in walking along a level road, up a hill, and home again: their pace on the level being 4 miles an hour, up hill 3, and down hill 6. Find distance walked: also (within half an hour) time of reaching top of hill.

Answer.—24 miles: half-past 6.

Solution.—A level mile takes ¼ of an hour, up hill ⅓, down hill ⅙. Hence to go and return over the same mile, whether on the level or on the hill-side, takes ½ an hour. Hence in 6 hours they went 12 miles out and 12 back. If the 12 miles out had been nearly all level, they would have taken a little over 3 hours; if nearly all up hill, a little under 4. Hence 3½ hours must be within ½ an hour of the time taken in reaching the peak; thus, as they started at 3, they got there within ½ an hour of ½ past 6.

Twenty-seven answers have come in. Of these, 9 are right, 16 partially right, and 2 wrong. The 16 give the *distance* correctly, but they have failed to grasp the fact that the top of the hill might have been reached at *any* moment between 6 o'clock and 7.

The two wrong answers are from Gerty Vernon and A Nihilist. The former makes the distance '23 miles,' while her revolutionary companion puts it at '27.' Gerty Vernon says 'they had to go 4 miles along the plain, and got to the foot of the hill at 4 o'clock.' They *might* have done so, I grant; but you have no ground for saying they *did* so. 'It was 7½ miles to the top of the hill, and they reached that at ¼ before 7 o'clock.' Here you go wrong in your arithmetic, and I must, however reluctantly, bid you farewell. 7½ miles, at 3 miles an hour, would *not* require 2¾ hours. A Nihilist says 'Let x denote the whole number of miles; y the number of hours to hill-top; ∴ $3y$ = number of miles to hill-top, and x-$3y$ = number of miles on the other side.' You bewilder me. The other side of *what*? 'Of the hill,' you say. But then, how did they get home again? However, to accommodate your views we will build a new hostelry at the foot of the hill on the opposite side, and also assume (what I grant you is *possible*, though it is not *necessarily* true) that there was no level road at all. Even then you go wrong.

You say

'$y = 6 - (x - 3y)/6$, ... (i);
$x/4\tfrac{1}{2} = 6$ ..... (ii).'

I grant you (i), but I deny (ii): it rests on the assumption that to go *part* of the time at 3 miles an hour, and the rest at 6 miles an hour, comes to the same result as going the *whole* time at 4½ miles an hour. But this would only be true if the '*part*' were an exact *half*, i.e., if they went up hill for 3 hours, and down hill for the other 3: which they certainly did *not* do.

The sixteen, who are partially right, are Agnes Bailey, F. K., Fifee, G. E. B., H. P., Kit, M. E. T., Mysie, A Mother's Son, Nairam, A Redruthian, A Socialist, Spear Maiden,

T. B. C, Vis Inertiæ, and Yak. Of these, F. K., Fifee, T. B. C, and Vis Inertiæ do not attempt the second part at all. F. K. and H. P. give no working. The rest make particular assumptions, such as that there was no level road—that there were 6 miles of level road—and so on, all leading to *particular* times being fixed for reaching the hill-top. The most curious assumption is that of Agnes Bailey, who says 'Let x = number of hours occupied in ascent; then $x/2$ = hours occupied in descent; and $4x/3$ = hours occupied on the level.' I suppose you were thinking of the relative *rates*, up hill and on the level; which we might express by saying that, if they went x miles up hill in a certain time, they would go $4x/3$ miles on the level *in the same time*. You have, in fact, assumed that they took *the same time* on the level that they took in ascending the hill. Fifee assumes that, when the aged knight said they had gone 'four miles in the hour' on the level, he meant that four miles was the *distance* gone, not merely the rate. This would have been—if Fifee will excuse the slang expression—a 'sell,' ill-suited to the dignity of the hero.

And now 'descend, ye classic Nine!' who have solved the whole problem, and let me sing your praises. Your names are Blithe, E. W., L. B., A Marlborough Boy, O. V. L., Putney Walker, Rose, Sea Breeze, Simple Susan, and Money Spinner. (These last two I count as one, as they send a joint answer.) Rose and Simple Susan and Co. do not actually state that the hill-top was reached some time between 6 and 7, but, as they have clearly grasped the fact that a mile, ascended and descended, took the same time as two level miles, I mark them as 'right.' A Marlborough Boy and Putney Walker deserve honourable mention for their algebraical solutions being the only two who have perceived that the question leads to *an indeterminate equation*. E. W. brings a charge of untruthfulness against the aged knight—a serious charge, for he was the very pink of chivalry! She says 'According to the data given, the time at the summit affords no clue to the total distance. It does not enable us to state precisely to an inch how much level and how much hill there was on the road.' 'Fair damsel,' the aged knight replies, '—if, as I surmise, thy initials denote Early Womanhood—bethink thee that the word "enable" is thine, not mine. I did but ask the time of reaching the hill-top as my *condition* for further parley. If *now* thou wilt not grant that I am a truth-loving man, then will I affirm that those same initials denote Envenomed Wickedness!'

CLASS LIST.

I.

A Marlborough Boy.

Putney Walker.

II.

Blithe.

E. W.

L. B.

O. V. L.

Rose.

Sea Breeze.

Simple Susan.

Money-Spinner.

Blithe has made so ingenious an addition to the problem, and Simple Susan and Co. have solved it in such tuneful verse, that I record both their answers in full. I have altered a word or two in Blithe's—which I trust she will excuse; it did not seem quite clear as it stood.

'Yet stay,' said the youth, as a gleam of inspiration lighted up the relaxing muscles of his quiescent features. 'Stay. Methinks it matters little *when* we reached that summit, the crown of our toil. For in the space of time wherein we clambered up one mile and bounded down the same on our return, we could have trudged the *twain* on the level. We have plodded, then, four-and-twenty miles in these six mortal hours; for never a moment did we stop for catching of fleeting breath or for gazing on the scene around!'

'Very good,' said the old man. 'Twelve miles out and twelve miles in. And we reached the top some time between six and seven of the clock. Now mark me! For every five minutes that had fled since six of the clock when we stood on yonder peak, so many miles had we toiled upwards on the dreary mountainside!'

The youth moaned and rushed into the hostel.

Blithe.

The elder and the younger knight,
They sallied forth at three;
How far they went on level ground
It matters not to me;
What time they reached the foot of hill,
When they began to mount,
Are problems which I hold to be
Of very small account.

The moment that each waved his hat
Upon the topmost peak—
To trivial query such as this
No answer will I seek.
Yet can I tell the distance well
They must have travelled o'er:
On hill and plain, 'twixt three and nine,
The miles were twenty-four.

Four miles an hour their steady pace
Along the level track,
Three when they climbed—but six when they
Came swiftly striding back
Adown the hill; and little skill
It needs, methinks, to show,
Up hill and down together told,
Four miles an hour they go.

For whether long or short the time
Upon the hill they spent,
Two thirds were passed in going up,

One third in the descent.
Two thirds at three, one third at six,
If rightly reckoned o'er,
Will make one whole at four—the tale
Is tangled now no more.

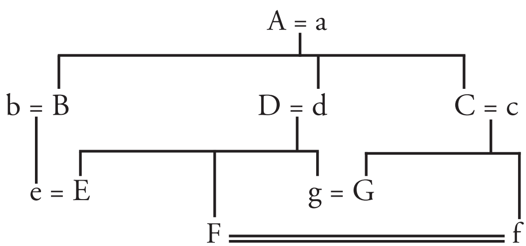
Simple Susan
Money Spinner

ANSWERS TO KNOT II.

§ 1. The Dinner Party.

Problem.—The Governor of Kgovjni wants to give a very small dinner party, and invites his father's brother-in-law, his brother's father-in-law, his father-in-law's brother, and his brother-in-law's father. Find the number of guests.

Answer.—One.



In this genealogy, males are denoted by capitals, and females by small letters.

The Governor is E and his guest is C.

Ten answers have been received. Of these, one is wrong, Galanthus Nivalis Major, who insists on inviting *two* guests, one being the Governor's *wife's brother's father*. If she had taken his *sister's husband's father* instead, she would have found it possible to reduce the guests to *one*.

Of the nine who send right answers, Sea-Breeze is the very faintest breath that ever bore the name! She simply states that the Governor's uncle might fulfill all the conditions 'by intermarriages'! 'Wind of the western sea,' you have had a very narrow escape! Be thankful to appear in the Class-list at all! Bog-Oak and Bradshaw of the Future use genealogies which require 16 people instead of 14, by inviting the Governor's *father's sister's husband* instead of his *father's wife's brother*. I cannot think this so good a solution as one that requires only 14. Caius and Valentine deserve special mention as the only two who have supplied genealogies.

CLASS LIST.

I.

Bee.

Caius.

M. M.

Matthew Matticks.

Old Cat.

Valentine.

II.

Bog-Oak.

Bradshaw of the Future.

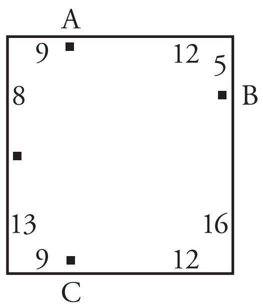
III.

Sea-Breeze.

§ 2. The Lodgings.

Problem.—A Square has 20 doors on each side, which contains 21 equal parts. They are numbered all round, beginning at one corner. From which of the four, Nos. 9, 25, 52, 73, is the sum of the distances, to the other three, least?

Answer.—From No. 9.



Let A be No. 9, B No. 25, C No. 52, and D No. 73.

$$\text{Then } AB = \sqrt{(12^2 + 5^2)} = \sqrt{169} = 13;$$
$$AC = 21;$$
$$AD = \sqrt{(9^2 + 8^2)} = \sqrt{145} = 12 +$$
(N.B. *i.e.* 'between 12 and 13.')
$$BC = \sqrt{(16^2 + 12^2)} = \sqrt{400} = 20;$$
$$BD = \sqrt{(3^2 + 21^2)} = \sqrt{450} = 21+;$$
$$CD = \sqrt{(9^2 + 13^2)} = \sqrt{250} = 15+;$$

Hence sum of distances from A is between 46 and 47; from B, between 54 and 55; from C, between 56 and 57; from D, between 48 and 51. (Why not 'between 48 and 49'? Make this out for yourselves.) Hence the sum is least for A.

Twenty-five solutions have been received. Of these, 15 must be marked '0,' 5 are partly right, and 5 right. Of the 15, I may dismiss Alphabetical Phantom, Bog-Oak, Dinah Mite, Fifee, Galanthus Nivalis Major (I fear the cold spring has blighted our Snowdrop), Guy, H.M.S. Pinafore, Janet, and Valentine with the simple remark that they insist on the unfortunate lodgers *keeping to the pavement*. (I used the words 'crossed to Number Seventy-three' for the special purpose of showing that *short cuts* were possible.) Sea-Breeze does the same, and adds that 'the result would be the same' even if they crossed the Square, but gives no proof of this. M. M. draws a diagram, and says that No. 9 is the house, 'as the diagram shows.' I cannot see *how* it does so. Old Cat assumes that the house *must* be No. 9 or No. 73. She does not explain how she estimates the distances. Bee's Arithmetic is faulty: she makes $\sqrt{169} + \sqrt{442} + \sqrt{130} = 741$. (I suppose you mean $\sqrt{741}$, which would be a little nearer the truth. But roots cannot be added in this manner. Do you think $\sqrt{9} + \sqrt{16}$ is 25, or even $\sqrt{25}$?) But Ayr's state is more perilous still: she draws illogical conclusions with a frightful calmness. After pointing out (rightly) that AC is less than BD she says, 'therefore the nearest house to the other three must be A or C.' And again, after pointing out (rightly) that B and D are both within the half-square containing A, she says 'therefore' AB + AD must be less than BC + CD. (There is no logical force in either 'therefore.' For the first, try Nos. 1, 21, 60, 70: this will make your premiss true, and your conclusion false. Similarly, for the second, try Nos. 1, 30, 51, 71.)

Of the five partly-right solutions, Rags and Tatters and

Mad Hatter (who send one answer between them) make No. 25 6 units from the corner instead of 5. Cheam, E. R. D. L., and Meggy Potts leave openings at the corners of the Square, which are not in the *data*: moreover Cheam gives values for the distances without any hint that they are only *approximations*. Crophi and Mophi make the bold and unfounded assumption that there were really 21 houses on each side, instead of 20 as stated by Balbus. 'We may assume,' they add, 'that the doors of Nos. 21, 42, 63, 84, are invisible from the centre of the Square'! What is there, I wonder, that Crophi and Mophi would *not* assume?

Of the five who are wholly right, I think Bradshaw of the Future, Caius, Clifton C., and Martreb deserve special praise for their full *analytical* solutions. Matthew Matticks picks out No. 9, and proves it to be the right house in two ways, very neatly and ingeniously, but *why* he picks it out does not appear. It is an excellent *synthetical* proof, but lacks the analysis which the other four supply.

CLASS LIST.

I.

Bradshaw of the Future.

Caius.

Clifton C.

Martreb.

II.

Matthew Matticks.

III.

Cheam.

Crophi and Mophi.

E. R. D. L.

Meggy Potts.

Rags and Tatters.

Mad Hatter.

A remonstrance has reached me from Scrutator on the subject of Knot I., which he declares was 'no problem at all.' 'Two questions,' he says, 'are put. To solve one there is no data: the other answers itself.' As to the first point, Scrutator is mistaken; there *are* (not 'is') data sufficient to answer the question. As to the other, it is interesting to know that the question 'answers itself,' and I am sure it does the question great credit: still I fear I cannot enter it on the list of winners, as this competition is only open to human beings.

ANSWERS TO KNOT III.

Problem.—(1) Two travellers, starting at the same time, went opposite ways round a circular railway. Trains start each way every 15 minutes, the easterly ones going round in 3 hours, the westerly in 2. How many trains did each meet on the way, not counting trains met at the terminus itself? (2) They went round, as before, each traveller counting as 'one' the train containing the other traveller. How many did each meet?

Answers.—(1) 19. (2) The easterly traveller met 12; the other 8.

The trains one way took 180 minutes, the other way 120. Let us take the L. C. M., 360, and divide the railway into 360 units. Then one set of trains went at the rate of 2 units a minute and at intervals of 30 units; the other at the rate of 3 units a minute and at intervals of 45 units. An easterly train starting has 45 units between it and the first train it will meet: it does $2/5$ of this while the other does $3/5$, and thus meets it at the end of 18 units, and so all the way round. A westerly train starting has 30 units between it and the first train it will meet: it does $3/5$ of this while the other does $2/5$, and thus meets it at the end of 18 units, and so all the way round. Hence if the railway be divided, by 19 posts, into 20 parts, each containing 18 units, trains meet at every post, and, in (1), each traveller passes 19 posts in going round, and so meets 19 trains. But, in (2), the easterly traveller only begins to count after traversing ⅖ of the journey, *i.e.*, on reaching the 8th post, and so counts 12 posts: similarly the other counts 8. They meet at the end of ⅖ of 3 hours, or ⅗ of 2 hours, *i.e.*, 72 minutes.

☼

Forty-five answers have been received. Of these 12 are beyond the reach of discussion, as they give no working. I can but enumerate their names. Ardmore, E. A., F. A. D., L. D., Matthew Matticks, M. E. T., Poo-Poo, and The Red Queen are all wrong. Beta and Rowena have got (1) right and (2) wrong. Cheeky Bob and Nairam give the right answers, but it may perhaps make the one less cheeky, and induce the other to take a less inverted view of things, to be informed that, if this had been a competition for a prize, they would have got no marks. [N.B.—I have not ventured to put E. A.'s name in full, as she only gave it provisionally, in case her answer should prove right.]

Of the 33 answers for which the working is given, 10 are wrong; 11 half-wrong and half-right; 3 right, except that they cherish the delusion that it was *Clara* who travelled in the easterly train—a point which the data do not enable us to settle; and 9 wholly right.

The 10 wrong answers are from Bo-Peep, Financier, I. W. T., Kate B., M. A. H., Q. Y. Z., Sea-Gull, Thistledown, Tom-Quad, and an unsigned one. Bo-Peep rightly says that the easterly traveller met all trains which started during the 3 hours of her trip, as well as all which started during the previous

2 hours, *i.e.*, all which started at the commencements of 20 periods of 15 minutes each; and she is right in striking out the one she met at the moment of starting; but wrong in striking out the *last* train, for she did not meet this at the terminus, but 15 minutes before she got there. She makes the same mistake in (2). Financier thinks that any train, met for the second time, is not to be counted. I. W. T. finds, by a process which is not stated, that the travellers met at the end of 71 minutes and 26½ seconds. Kate B. thinks the trains which are met on starting and on arriving are *never* to be counted, even when met elsewhere. Q. Y. Z. tries a rather complex algebraical solution, and succeeds in finding the time of meeting correctly: all else is wrong. Sea-Gull seems to think that, in (1), the easterly train *stood still* for 3 hours; and says that, in (2), the travellers met at the end of 71 minutes 40 seconds. Thistledown nobly confesses to having tried no calculation, but merely having drawn a picture of the railway and counted the trains; in (1), she counts wrong; in (2) she makes them meet in 75 minutes. Tom-Quad omits (1): in (2) he makes Clara count the train she met on her arrival. The unsigned one is also unintelligible; it states that the travellers go '1/24 more than the total distance to be traversed'! The 'Clara' theory, already referred to, is adopted by 5 of these, viz., Bo-Peep, Financier, Kate B., Tom-Quad, and the nameless writer.

The 11 half-right answers are from Bog-Oak, Bridget, Castor, Cheshire Cat, G. E. B., Guy, Mary, M. A. H., Old Maid, R. W., and Vendredi. All these adopt the 'Clara' theory. Castor omits (1). Vendredi gets (1) right, but in (2) makes the same mistake as Bo-Peep. I notice in your solution a marvellous proportion-sum:—'300 miles: 2 hours :: one mile: 24 seconds.'

May I venture to advise your acquiring, as soon as possible, an utter disbelief in the possibility of a ratio existing between *miles* and *hours*? Do not be disheartened by your two friends' sarcastic remarks on your 'roundabout ways.' Their short method, of adding 12 and 8, has the slight disadvantage of bringing the answer wrong: even a 'roundabout' method is better than *that*! M. A. H., in (2), makes the travellers count 'one' *after* they met, not *when* they met. Cheshire Cat and Old Maid get '20' as answer for (1), by forgetting to strike out the train met on arrival. The others all get '18' in various ways. Bog-Oak, Guy, and R. W. divide the trains which the westerly traveller has to meet into 2 sets, viz., those already on the line, which they (rightly) make '11,' and those which started during her 2 hours' journey (exclusive of train met on arrival), which they (wrongly) make '7'; and they make a similar mistake with the easterly train. Bridget (rightly) says that the westerly traveller met a train every 6 minutes for 2 hours, but (wrongly) makes the number '20'; it should be '21.' G. E. B. adopts Bo-Peep's method, but (wrongly) strikes out (for the easterly traveller) the train which started at the *commencement* of the previous 2 hours. Mary thinks a train, met on arrival, must not be counted, even when met on a *previous* occasion.

The 3, who are wholly right but for the unfortunate 'Clara' theory, are F. Lee, G. S. C., and X. A. B.

And now 'descend, ye classic Ten!' who have solved the whole problem. Your names are Aix-les-Bains, Algernon Bray (thanks for a friendly remark, which comes with a heart-warmth that not even the Atlantic could chill), Arvon, Bradshaw of the Future, Fifee, H. L. R., J. L. O., Omega, S. S. G., and Waiting for the Train. Several of these have put Clara, provisionally, into the easterly train: but they seem to have understood that the data do not decide that point.

CLASS LIST.

I.

Aix-les-Bains.

Algernon Bray.

Bradshaw of the Future.

Fifee.

H. L. R.

Omega.

S. S. G.

Waiting for the train.

II.

Arvon.

J. L. O.

III.

F. Lee.

G. S. C.

X. A. B.

ANSWERS TO KNOT IV.

Problem.—There are 5 sacks, of which Nos. 1, 2, weigh 12 lbs.; Nos. 2, 3, 13½ lbs.; Nos. 3, 4, 11½ lbs.; Nos. 4, 5, 8 lbs.; Nos. 1, 3, 5, 16 lbs. Required the weight of each sack.

Answer.—5½, 6½, 7, 4½, 3½.

The sum of all the weighings, 61 lbs., includes sack No. 3 *thrice* and each other *twice*. Deducting twice the sum of the 1st and 4th weighings, we get 21 lbs. for *thrice* No. 3, *i.e.*, 7 lbs. for No. 3. Hence, the 2nd and 3rd weighings give 6½ lbs., 4½ lbs. for Nos. 2, 4; and hence again, the 1st and 4th weighings give 5½ lbs., 3½ lbs., for Nos. 1, 5.

Ninety-seven answers have been received. Of these, 15 are beyond the reach of discussion, as they give no working. I can but enumerate their names, and I take this opportunity of saying that this is the last time I shall put on record the names of competitors who give no sort of clue to the process by which their answers were obtained. In guessing a conundrum, or in catching a flea, we do not expect the breathless victor to give us afterwards, in cold blood, a history of the mental or muscular efforts by which he achieved success; but a mathematical calculation is another thing.

The names of this 'mute inglorious' band are Common Sense, D. E. R., Douglas, E. L., Ellen, I. M. T., J. M. C., Joseph, Knot I, Lucy, Meek, M. F. C., Pyramus, Shah, Veritas.

Of the eighty-two answers with which the working, or some approach to it, is supplied, one is wrong: seventeen have given solutions which are (from one cause or another) practically valueless: the remaining sixty-four I shall try to arrange in a Class-list, according to the varying degrees of shortness and neatness to which they seem to have attained.

The solitary wrong answer is from Nell. To be thus 'alone in the crowd' is a distinction—a painful one, no doubt, but still a distinction. I am sorry for you, my dear young lady, and I seem to hear your tearful exclamation, when you read these lines, 'Ah! This is the knell of all my hopes!' Why, oh why, did you assume that the 4th and 5th bags weighed 4 lbs. each? And why did you not test your answers? However, please try again: and please don't change your *nom-de-plume*: let us have Nell in the First Class next time!

The seventeen whose solutions are practically valueless are Ardmore, A Ready Reckoner, Arthur, Bog-Lark, Bog-Oak, Bridget, First Attempt, J. L. C., M. E. T., Rose, Rowena, Sea-Breeze, Sylvia, Thistledown, Three-Fifths Asleep, Vendredi, and Winifred. Bog-Lark tries it by a sort of 'rule of false,' assuming experimentally that Nos. 1, 2, weigh 6 lbs. each, and having thus produced 17½, instead of 16, as the weight of 1, 3, and 5, she removes 'the superfluous pound and a half,' but does not explain how she knows from which to take it. Three-Fifths Asleep says that (when in that peculiar state) 'it seemed perfectly clear' to her that, '3 out of the 5 sacks being weighed twice over, ⅗ of 45 = 27, must be the total weight of the 5 sacks.' As to which I can only say, with the Captain, 'it beats me entirely!' Winifred, on the plea that 'one must have a starting-point,' assumes (what I fear is a mere guess) that No. 1 weighed 5½ lbs. The rest all do it, wholly or partly, by guess-work.

The problem is of course (as any Algebraist sees at once) a case of 'simultaneous simple equations.' It is, however, easily soluble by Arithmetic only; and, when this is the case, I hold that it is bad workmanship to use the more complex method. I have not, this time, given more credit to arithmetical solutions; but in future problems I shall (other things being equal) give the highest marks to those who use the simplest machinery. I have put into Class I. those whose answers seemed specially short and neat, and into Class III. those that seemed specially long or clumsy. Of this last set, A. C. M., Furze-Bush, James, Partridge, R. W., and Waiting for the Train, have sent long wandering solutions, the substitutions having no definite method, but seeming to have been made to see what would come of it. Chilpome and Dublin Boy omit some of the working. Arvon Marlborough Boy only finds the weight of *one* sack.

CLASS LIST.

I.

B. E. D.

C. H.

Constance Johnson.

Greystead.

Guy.

Hoopoe.

J. F. A.

M. A. H.

Number Five.

Pedro.

R. E. X.

Seven Old Men.

Vis Inertiæ.

Willy B.

Yahoo.

II.

American Subscriber.

An Appreciative Schoolma'am.

Ayr.

Bradshaw of the Future.

Cheam.

C. M. G.

Dinah Mite.

Duckwing.

E. C. M.

E. N. Lowry.

Era.

Euroclydon.

F. H. W.

Fifee.

G. E. B.

Harlequin.

Hawthorn.

Hough Green.

J. A. B.

Jack Tar.

J. B. B.

Kgovjni.

Land Lubber.

L. D.

Magpie.

Mary.

Mhruxi.

Minnie.

Money-Spinner.

Nairam.

Old Cat.

Polichinelle.

Simple Susan.

S. S. G.

Thisbe.

Verena.

Wamba.

Wolfe.

Wykehamicus.

Y. M. A. H.

III.

A. C. M.

Arvon Marlborough Boy.

Chilpome.

Dublin Boy.

Furze-Bush.

James.

Partridge.

R. W.

Waiting for the Train.

ANSWERS TO KNOT V.

Problem.—To mark pictures, giving 3 x's to 2 or 3, 2 to 4 or 5, and 1 to 9 or 10; also giving 3 o's to 1 or 2, 2 to 3 or 4 and 1 to 8 or 9; so as to mark the smallest possible number of pictures, and to give them the largest possible number of marks.

Answer.—10 pictures; 29 marks; arranged thus:—

x	x	x	x	x	x	x	x	x	o
x	x	x	x	x		o	o	o	o
x	x	o	o	o	o	o	o	o	o

Solution.—By giving all the x's possible, putting into brackets the optional ones, we get 10 pictures marked thus:—

x	x	x	x	x	x	x	x	x	(x)
x	x	x	x	(x)					
x	x	(x)							

By then assigning o's in the same way, beginning at the other end, we get 9 pictures marked thus:—

							(o)	o
					(o)	o	o	o
(o)	o	o	o	o	o	o	o	o

All we have now to do is to run these two wedges as close together as they will go, so as to get the minimum number of pictures—erasing optional marks where by so doing we can run them closer, but otherwise letting them stand. There are 10 necessary marks in the 1st row, and in the 3rd; but only 7 in the 2nd. Hence we erase all optional marks in the 1st and 3rd rows, but let them stand in the 2nd.

Twenty-two answers have been received. Of these 11 give no working; so, in accordance with what I announced in my last review of answers, I leave them unnamed, merely mentioning that 5 are right and 6 wrong.

Of the eleven answers with which some working is supplied, 3 are wrong. C. H. begins with the rash assertion that under the given conditions 'the sum is impossible. For,' he or she adds (these initialed correspondents are dismally vague beings to deal with: perhaps 'it' would be a better pronoun), '10 is the least possible number of pictures' (granted): 'therefore we must either give 2 x's to 6, or 2 o's to 5.' Why 'must,' oh alphabetical phantom? It is nowhere ordained that every picture 'must' have 3 marks! Fifee sends a folio page of solution, which deserved a better fate: she offers 3 answers, in each of which 10 pictures are marked, with 30 marks; in one she gives 2 x's to 6 pictures; in another to 7; in the 3rd she gives 2 o's to 5; thus in every case ignoring the conditions. (I pause to remark that the condition '2 x's to 4 or 5 pictures' can only mean '*either* to 4 *or else* to 5': if, as one competitor holds, it might mean *any* number not less than 4, the words '*or* 5' would be superfluous.) I. E. A. (I am happy to say that none of these bloodless phantoms appear this time in the class-list. Is it IDEA with the 'D' left out?) gives 2 x's to 6 pictures. She then takes me to task for using the word 'ought' instead of 'nought.' No doubt, to one who thus rebels against the rules laid down for her guidance, the word must be distasteful. But does not I. E. A. remember the parallel case of 'adder'? That creature was originally 'a nadder': then the two words took to bandying the poor 'n' backwards and forwards like a shuttlecock, the final state of the game being 'an adder.' May not 'a nought' have similarly become 'an ought'? Anyhow, 'oughts and crosses' is a very old game. I don't think I ever heard it called 'noughts and crosses.'

In the following Class-list, I hope the solitary occupant of III. will sheathe her claws when she hears how narrow an escape she has had of not being named at all. Her account of the process by which she got the answer is so meagre that, like the nursery tale of 'Jack-a-Minory' (I trust I. E. A. will be merciful to the spelling), it is scarcely to be distinguished from 'zero.'

CLASS LIST.

I.

Guy.

Old Cat.

Sea-Breeze.

II.

Ayr.

Bradshaw of the Future.

F. Lee.

H. Vernon.

III.

Cat.

ANSWERS TO KNOT VI.

Problem 1.—*A* and *B* began the year with only 1,000£ a-piece. They borrowed nought; they stole nought. On the next New-Year's Day they had 60,000£ between them. How did they do it?

Solution.—They went that day to the Bank of England. *A* stood in front of it, while *B* went round and stood behind it.

Two answers have been received, both worthy of much honour. Addlepate makes them borrow '0' and steal '0,' and uses both cyphers by putting them at the right-hand end of the 1,000£, thus producing 100,000£, which is well over the mark. But (or to express it in Latin) At Spes Infracta has solved it even more ingeniously: with the first cypher she turns the '1' of the 1,000£ into a '9,' and adds the result to the original sum, thus getting 10,000£: and in this, by means of the other '0,' she turns the '1' into a '6,' thus hitting the exact 60,000£.

CLASS LIST

I.

At Spes Infracta.

II.

Addlepate.

Problem 2.—*L* makes 5 scarves, while *M* makes 2: *Z* makes 4 while *L* makes 3. Five scarves of *Z*'s weigh one of *L*'s; 5 of *M*'s weigh 3 of *Z*'s. One of *M*'s is as warm as 4 of *Z*'s: and one of *L*'s as warm as 3 of *M*'s. Which is best, giving equal weight in the result to rapidity of work, lightness, and warmth?

Answer.—The order is *M*, *L*, *Z*.

Solution.—As to rapidity (other things being constant) *L*'s merit is to *M*'s in the ratio of 5 to 2: *Z*'s to *L*'s in the ratio of 4 to 3. In order to get one set of 3 numbers fulfilling these conditions, it is perhaps simplest to take the one that occurs *twice* as unity, and reduce the others to fractions: this gives, for *L*, *M*, and *Z*, the marks 1, ⅖, ⅔. In estimating for *lightness*, we observe that the greater the weight, the less the merit, so that *Z*'s merit is to *L*'s as 5 to 1. Thus the marks for *lightness* are ⅕, ⅔, 1. And similarly, the marks for warmth are 3, 1, ¼. To get the total result, we must *multiply* *L*'s 3 marks together, and do the same for *M* and for *Z*. The final numbers are 1 × ⅕ × 3, ⅖ × ⅔ × 1, ⅔ × 1 × ¼; *i.e.* ⅗, ⅔, ⅓; *i.e.* multiplying throughout by 15 (which will not alter the proportion), 9, 10, 5; showing the order of merit to be *M*, *L*, *Z*.

Twenty-nine answers have been received, of which five are right, and twenty-four wrong. These hapless ones have all (with three exceptions) fallen into the error of *adding* the proportional numbers together, for each candidate, instead of *multiplying*. *Why* the latter is right, rather than the former, is fully proved in text-books, so I will not occupy space by stating it here: but it can be *illustrated* very easily by the case of length, breadth, and depth. Suppose *A* and *B* are rival diggers of rectangular tanks: the amount of work done is evidently measured by the number of *cubical feet* dug out. Let *A* dig a tank 10 feet long, 10 wide, 2 deep: let *B* dig one 6 feet long, 5 wide, 10 deep. The cubical contents are 200, 300; *i.e. B* is best digger in the ratio of 3 to 2. Now try marking for length, width, and depth, separately; giving a maximum mark of 10 to the best in each contest, and then *adding* the results!

Of the twenty-four malefactors, one gives no working, and so has no real claim to be named; but I break the rule for once, in deference to its success in Problem 1: he, she, or it, is Addlepate. The other twenty-three may be divided into five groups.

First and worst are, I take it, those who put the rightful winner *last*; arranging them as 'Lolo, Zuzu, Mimi.' The names of these desperate wrong-doers are Ayr, Bradshaw of the Future, Furze-bush and Pollux (who send a joint answer), Greystead, Guy, Old Hen, and Simple Susan. The latter was *once* best of all; the Old Hen has taken advantage of her simplicity, and beguiled her with the chaff which was the bane of her own chickenhood.

Secondly, I point the finger of scorn at those who have put the worst candidate at the top; arranging them as 'Zuzu, Mimi, Lolo.' They are Graecia, M. M., Old Cat, and R. E. X. ''Tis Greece, but—.'

The third set have avoided both these enormities, and have even succeeded in putting the worst last, their answer being 'Lolo, Mimi, Zuzu.' Their names are Ayr (who also appears among the 'quite too too'), Clifton C., F. B., Fifee, Grig, Janet, and Mrs. Sairey Gamp. F. B. has not fallen into the common error; she *multiplies* together the proportionate numbers she gets, but in getting them she goes wrong, by reckoning warmth as a *de*-merit. Possibly she is 'Freshly Burnt,' or comes 'From Bombay.' Janet and Mrs. Sairey Gamp have also avoided this error: the method they have adopted is shrouded in mystery—I scarcely feel competent to criticize it. Mrs. Gamp says 'if Zuzu makes 4 while Lolo makes 3, Zuzu makes 6 while Lolo makes 5 (bad reasoning), while Mimi makes 2.' From this she concludes 'therefore Zuzu excels in speed by 1' (*i.e.* when compared with Lolo; but what about Mimi?). She then compares the 3 kinds of excellence, measured on this mystic scale. Janet takes the statement, that 'Lolo makes 5 while Mimi makes 2,' to prove that 'Lolo makes 3 while Mimi makes 1 and Zuzu 4' (worse reasoning than Mrs. Gamp's), and thence concludes that 'Zuzu excels in speed by ⅛'! Janet should have been Adeline, 'mystery of mysteries!'

The fourth set actually put Mimi at the top, arranging them as 'Mimi, Zuzu, Lolo.' They are Marquis and Co., Martreb, S. B. B. (first initial scarcely legible: *may* be meant for 'J'), and Stanza.

The fifth set consist of An ancient Fish and Camel. These ill-assorted comrades, by dint of foot and fin, have scrambled into the right answer, but, as their method is wrong, of course it counts for nothing. Also An ancient Fish has very ancient and fishlike ideas as to *how* numbers represent merit: she says 'Lolo gains 2½ on Mimi.' Two and a half *what*? Fish, fish, art thou in thy duty?

Of the five winners I put Balbus and The elder Traveller slightly below the other three—Balbus for defective reasoning, the other for scanty working. Balbus gives two reasons for saying that *addition* of marks is *not* the right method, and then adds 'it follows that the decision must be made by *multiplying* the marks together.' This is hardly more logical than to say 'This is not Spring: *therefore* it must be Autumn.'

CLASS LIST.

I.

Dinah Mite.

E. B. D. L.

Joram.

II.

Balbus.

The Elder Traveller.

With regard to Knot V., I beg to express to Vis Inertiæ and to any others who, like her, understood the condition to be that *every* marked picture must have *three* marks, my sincere regret that the unfortunate phrase '*fill* the columns with oughts and crosses' should have caused them to waste so much time and trouble. I can only repeat that a *literal* interpretation of 'fill' would seem to *me* to require that *every* picture in the gallery should be marked. Vis Inertiæ would have been in the First Class if she had sent in the solution she now offers.

ANSWERS TO KNOT VII.

Problem.—Given that one glass of lemonade, 3 sandwiches, and 7 biscuits, cost 1*s.* 2*d.*; and that one glass of lemonade, 4 sandwiches, and 10 biscuits, cost 1*s.* 5*d.*: find the cost of (1) a glass of lemonade, a sandwich, and a biscuit; and (2) 2 glasses of lemonade, 3 sandwiches, and 5 biscuits.

Answer.—(1) 8*d.*; (2) 1*s.* 7*d.*

Solution.—This is best treated algebraically. Let x = the cost (in pence) of a glass of lemonade, y of a sandwich, and z of a biscuit. Then we have $x + 3y + 7z = 14$, and $x + 4y + 10z = 17$. And we require the values of $x + y + z$, and of $2x + 3y + 5z$. Now, from *two* equations only, we cannot find, *separately*, the values of *three* unknowns: certain *combinations* of them may, however, be found. Also we know that we can, by the help of the given equations, eliminate 2 of the 3 unknowns from the quantity whose value is required, which will then contain one only. If, then, the required value is ascertainable at all, it can only be by the 3rd unknown vanishing of itself: otherwise the problem is impossible.

Let us then eliminate lemonade and sandwiches, and reduce everything to biscuits—a state of things even more depressing than 'if all the world were apple-pie'—by subtracting the 1st equation from the 2nd, which eliminates lemonade, and gives $y + 3z = 3$, or $y = 3-3z$; and then substituting this value of y in the 1st, which gives $x-2z = 5$, *i.e.* $x = 5 + 2z$. Now if we substitute these values of x, y, in the quantities whose values are required, the first becomes $(5 + 2z) + (3-3z) + z$, *i.e.* 8: and the second becomes $2(5 + 2z) + 3(3\text{-}3z) + 5z$, *i.e.* 19. Hence the answers are (1) 8*d.*, (2) 1*s.* 7*d.*

☼

The above is a *universal* method: that is, it is absolutely certain either to produce the answer, or to prove that no answer is possible. The question may also be solved by combining the quantities whose values are given, so as to form those whose values are required. This is merely a matter of ingenuity and good luck: and as it *may* fail, even when the thing is possible, and is of no use in proving it *im*possible, I cannot rank this method as equal in value with the other. Even when it succeeds, it may prove a very tedious process. Suppose the 26 competitors, who have sent in what I may call *accidental* solutions, had had a question to deal with where every number contained 8 or 10 digits! I suspect it would have been a case of 'silvered is the raven hair' (see 'Patience') before any solution would have been hit on by the most ingenious of them.

Forty-five answers have come in, of which 44 give, I am happy to say, some sort of *working*, and therefore deserve to be mentioned by name, and to have their virtues, or vices as the case may be, discussed. Thirteen have made assumptions to which they have no right, and so cannot figure in the Class-list, even though, in 10 of the 13 cases, the answer is right. Of the remaining 28, no less than 26 have sent in *accidental* solutions, and therefore fall short of the highest honours.

I will now discuss individual cases, taking the worst first, as my custom is.

FROGGY gives no working—at least this is all he gives: after stating the given equations, he says 'therefore the difference, 1 sandwich + 3 biscuits, = 3*d.*': then follow the amounts of the unknown bills, with no further hint as to how he got them. Froggy has had a *very* narrow escape of not being named at all!

Of those who are wrong, Vis Inertiæ has sent in a piece of incorrect working. Peruse the horrid details, and shudder! She takes x (call it 'y') as the cost of a sandwich, and concludes (rightly enough) that a biscuit will cost $(3\text{-}y)/3$. She then subtracts the second equation from the first, and deduces $3y + 7 \times (3\text{-}y)/3\text{-}4y + 10 \times (3\text{-}y)/3 = 3$. By making two mistakes in this line, she brings out $y = 3/2$. Try it again, oh Vis Inertiæ! Away with Inertiæ: infuse a little more Vis: and you will bring out the correct (though uninteresting) result, $0 = 0$! This will show you that it is hopeless to try to coax any one of these 3 unknowns to reveal its *separate* value. The other competitor, who is wrong throughout, is either J. M. C. or T. M. C.: but, whether he be a Juvenile Mis-Calculator or a True Mathematician Confused, he makes the answers 7*d.* and 1*s.* 5*d.* He assumes, with Too Much Confidence, that biscuits were ½*d.* each, and that Clara paid for 8, though she only ate 7!

We will now consider the 13 whose working is wrong, though the answer is right: and, not to measure their demerits too exactly, I will take them in alphabetical order. Anita finds (rightly) that '1 sandwich and 3 biscuits cost 3*d.*,' and proceeds 'therefore 1 sandwich = 1½*d.*, 3 biscuits = 1½*d.*, 1 lemonade = 6*d.*' Dinah Mite begins like Anita: and thence proves (rightly) that a biscuit costs less than a 1*d.*: whence she concludes (wrongly) that it *must* cost ½*d.* F. C. W. is so beautifully resigned to the certainty of a verdict of 'guilty,' that I have hardly the heart to utter the word, without adding a 'recommended to mercy owing to extenuating circumstances.' But really, you know, where *are* the extenuating circumstances? She begins by assuming that lemonade is 4*d.* a glass, and sandwiches 3*d.* each, (making with the 2 given equations, *four* conditions to be fulfilled by *three* miserable unknowns!). And, having (naturally) developed this into a contradiction, she then tries 5*d.* and 2*d.* with a similar result. (N.B.—*This* process might have been carried on through the whole of the Tertiary Period, without gratifying one single Megatherium.) She then, by a 'happy thought,' tries half-penny biscuits, and so obtains a consistent result. This may be a good solution, viewing the problem as a conundrum: but it is *not* scientific. JANET identifies sandwiches with biscuits! 'One sandwich + 3 biscuits' she makes equal to '4.' Four *what*? MAYFAIR makes the astounding assertion that the equation, $s + 3b = 3$, 'is evidently only satisfied by $s = 2/2$, $b = 1/2$'! OLD CAT believes that the assumption that a sandwich costs 1½*d.* is 'the only way to avoid unmanageable fractions.' But *why* avoid them? Is there not a certain glow of triumph in taming such a fraction? 'Ladies and gentlemen, the fraction now before you is one that for years defied all efforts of a refining nature: it was, in a word, hopelessly vulgar. Treating it as a circulating decimal (the treadmill of fractions) only made matters worse. As a last resource, I reduced it to its lowest terms, and extracted its square root!' Joking apart, let me thank OLD CAT for some very kind words of sympathy, in reference to a correspondent (whose name I am happy to say I have now forgotten) who had found fault with me as a discourteous critic. O. V. L. is beyond my comprehension. He takes the given equations as (1) and (2): thence, by the process [(2)-(1)] deduces (rightly) equation (3) viz. $s + 3b = 3$: and thence again, by the process [x3] (a hopeless mystery), deduces $3s + 4b$

= 4. I have nothing to say about it: I give it up. Sea-Breeze says 'it is immaterial to the answer' (why?) 'in what proportion 3*d.* is divided between the sandwich and the 3 biscuits': so she assumes $s = 1\frac{1}{2}d.$, $b = \frac{1}{2}d.$ Stanza is one of a very irregular metre. At first she (like Janet) identifies sandwiches with biscuits. She then tries two assumptions ($s = 1$, $b = \frac{2}{3}$, and $s = \frac{1}{2}$ $b = \frac{2}{6}$), and (naturally) ends in contradictions. Then she returns to the first assumption, and finds the 3 unknowns separately: *quod est absurdum*. Stiletto identifies sandwiches and biscuits, as 'articles.' Is the word ever used by confectioners? I fancied 'What is the next article, Ma'am?' was limited to linendrapers. Two Sisters first assume that biscuits are 4 a penny, and then that they are 2 a penny, adding that 'the answer will of course be the same in both cases.' It is a dreamy remark, making one feel something like Macbeth grasping at the spectral dagger. 'Is this a statement that I see before me?' If you were to say 'we both walked the same way this morning,' and *I* were to say '*one* of you walked the same way, but the other didn't,' which of the three would be the most hopelessly confused? Turtle Pyate (what *is* a Turtle Pyate, please?) and Old Crow, who send a joint answer, and Y. Y., adopt the same method. Y. Y. gets the equation $s + 3b = 3$: and then says 'this sum must be apportioned in one of the three following ways.' It *may* be, I grant you: but Y. Y. do you say 'must'? I fear it is *possible* for Y. Y. to be *two* Y's. The other two conspirators are less positive: they say it 'can' be so divided: but they add 'either of the three prices being right'! This is bad grammar and bad arithmetic at once, oh mysterious birds!

Of those who win honours, The Shetland Snark must have the 3rd class all to himself. He has only answered half the question, viz. the amount of Clara's luncheon: the two little old ladies he pitilessly leaves in the midst of their 'difficulty.' I beg to assure him (with thanks for his friendly remarks) that entrance-fees and subscriptions are things unknown in that most economical of clubs, 'The Knot-Untiers.'

The authors of the 26 'accidental' solutions differ only in the number of steps they have taken between the *data* and the answers. In order to do them full justice I have arranged the 2nd class in sections, according to the number of steps. The two Kings are fearfully deliberate! I suppose walking quick, or taking short cuts, is inconsistent with kingly dignity: but really, in reading Theseus' solution, one almost fancied he was 'marking time,' and making no advance at all! The other King will, I hope, pardon me for having altered 'Coal' into 'Cole.' King Coilus, or Coil, seems to have reigned soon after Arthur's time. Henry of Huntingdon identifies him with the King Coël who first built walls round Colchester, which was named after him. In the Chronicle of Robert of Gloucester we read:—

> *Aftur Kyng Aruirag, of wam we habbeth y told,*
> *Marius ys sone was kyng, quoynte mon & bold.*
> *And ys sone was aftur hym,* Coil *was ys name,*
> *Bothe it were quoynte men, & of noble fame.*

Balbus lays it down as a general principle that 'in order to ascertain the cost of any one luncheon, it must come to the same amount upon two different assumptions.' (*Query.* Should not 'it' be 'we'? Otherwise the *luncheon* is represented as wishing to ascertain its own cost!) He then makes two assumptions—one, that sandwiches cost nothing; the other, that biscuits cost nothing, (either arrangement would lead to the shop being inconveniently crowded!)—and brings out the unknown luncheons as 8*d.* and 19*d.*, on each assumption. He then concludes that this agreement of results 'shows that the answers are correct.' Now I propose to disprove his general law by simply giving *one* instance of its failing. One instance is quite enough. In logical language, in order to disprove a 'universal affirmative,' it is enough to prove its contradictory, which is a 'particular negative.' (I must pause for a digression on Logic, and especially on Ladies' Logic. The universal affirmative 'everybody says he's a duck' is crushed instantly by proving the particular negative 'Peter says he's a goose,' which is equivalent to 'Peter does *not* say he's a duck.' And the universal negative 'nobody calls on her' is well met by the particular affirmative '*I* called yesterday.' In short, either of two contradictories disproves the other: and the moral is that, since a particular proposition is much more easily proved than a universal one, it is the wisest course, in arguing with a Lady, to limit one's *own* assertions to 'particulars,' and leave *her* to prove the 'universal' contradictory, if she can. You will thus generally secure a *logical* victory: a *practical* victory is not to be hoped for, since she can always fall back upon the crushing remark '*that* has nothing to do with it!'—a move for which Man has not yet discovered any satisfactory answer. Now let us return to Balbus.) Here is my 'particular negative,' on which to test his rule. Suppose the two recorded luncheons to have been '2 buns, one queen-cake, 2 sausage-rolls, and a bottle of Zoëdone: total, one-and-ninepence,' and 'one bun, 2 queen-cakes, a sausage-roll, and a bottle of Zoëdone: total, one-and-fourpence.' And suppose Clara's unknown luncheon to have been '3 buns, one queen-cake, one sausage-roll, and 2 bottles of Zoëdone:' while the two little sisters had been indulging in '8 buns, 4 queen-cakes, 2 sausage-rolls, and 6 bottles of Zoëdone.' (Poor souls, how thirsty they must have been!) If Balbus will kindly try this by his principle of 'two assumptions,' first assuming that a bun is 1*d.* and a queen-cake 2*d.*, and then that a bun is 3*d.* and a queen-cake 3*d.*, he will bring out the other two luncheons, on each assumption, as 'one-and-nine-pence' and 'four-and-ten-pence' respectively, which harmony of results, he will say, 'shows that the answers are correct.' And yet, as a matter of fact, the buns were 2*d.* each, the queen-cakes 3*d.*, the sausage-rolls 6*d.*, and the Zoëdone 2*d.* a bottle: so that Clara's third luncheon had cost one-and-sevenpence, and her thirsty friends had spent four-and-fourpence!

Another remark of Balbus I will quote and discuss: for I think that it also may yield a moral for some of my readers. He says 'it is the same thing in substance whether in solving this problem we use words and call it Arithmetic, or use letters and signs and call it Algebra.' Now this does not appear to me a correct description of the two methods: the Arithmetical method is that of 'synthesis' only; it goes from one known fact to another, till it reaches its goal: whereas the Algebraical method is that of 'analysis': it begins with the goal, symbolically represented, and so goes backwards, dragging its veiled victim with it, till it has reached the full daylight of known facts, in which it can tear off the veil and say 'I know you!'

Take an illustration. Your house has been broken into and robbed, and you appeal to the policeman who was on duty that night. 'Well, Mum, I did see a chap getting out over your garden-wall: but I was a good bit off, so I didn't chase him, like. I just cut down the short way to the Chequers, and who should I meet but Bill Sykes, coming full split round the corner. So I just ups and says “My lad, you’re wanted.” That’s all I says. And he says “I’ll go along quiet, Bobby,” he says, “without the darbies,” he says.’ There’s your *Arithmetical* policeman. Now try the other method. ‘I seed somebody a running, but he was well gone or ever *I* got nigh the place. So I just took a look round in the garden. And I noticed the foot-marks, where the chap had come right across your flower-beds. They was good big foot-marks sure-ly. And I noticed as the left foot went down at the heel, ever so much deeper than the other. And I says to myself “The chap’s been a big hulking chap: and he goes lame on his left foot.” And I rubs my hand on the wall where he got over, and there was soot on it, and no mistake. So I says to myself “Now where can I light on a big man, in the chimbley-sweep line, what’s lame of one foot?” And I flashes up permiscuous: and I says “It’s Bill Sykes!” says I.’ There is your *Algebraical* policeman—a higher intellectual type, to my thinking, than the other.

LITTLE JACK’s solution calls for a word of praise, as he has written out what really is an algebraical proof *in words*, without representing any of his facts as equations. If it is all his own, he will make a good algebraist in the time to come. I beg to thank SIMPLE SUSAN for some kind words of sympathy, to the same effect as those received from OLD CAT.

HECLA and MARTREB are the only two who have used a method *certain* either to produce the answer, or else to prove it impossible: so they must share between them the highest honours.

CLASS LIST.

I.

Hecla.

Martreb.

II.

§ 1 (2 *steps*).

Adelaide.

Clifton C....

E. K. C.

Guy.

L'Inconnu.

Little Jack.

Nil Desperandum.

Simple Susan.

Yellow-Hammer.

Woolly One.

§ 2 (3 *steps*).

A. A.

A Christmas Carol.

Afternoon Tea.

An appreciative Schoolma'am.

Baby.

Balbus.

Bog-Oak.

The Red Queen.

Wall-Flower.

§ 3 (4 *steps*).

Hawthorn.

Joram.

S. S. G.

§ 4 (5 *steps*).

A Stepney Coach.

§ 5 (6 *steps*).

Bay Laurel.

Bradshaw of the Future.

§ 6 (9 *steps*).

Old King Cole.

§ 7 (14 *steps*).

Theseus.

ANSWERS TO CORRESPONDENTS.

I have received several letters on the subjects of Knots II. and VI., which lead me to think some further explanation desirable.

In Knot II., I had intended the numbering of the houses to begin at one corner of the Square, and this was assumed by most, if not all, of the competitors. Trojanus however says 'assuming, in default of any information, that the street enters the square in the middle of each side, it may be supposed that the numbering begins at a street.' But surely the other is the more natural assumption?

In Knot VI., the first Problem was of course a mere *jeu de mots*, whose presence I thought excusable in a series of Problems whose aim is to entertain rather than to instruct: but it has not escaped the contemptuous criticisms of two of my correspondents, who seem to think that Apollo is in duty bound to keep his bow always on the stretch. Neither of them has guessed it: and this is true human nature. Only the other day—the 31st of September, to be quite exact—I met my old friend Brown, and gave him a riddle I had just heard. With one great effort of his colossal mind, Brown guessed it. 'Right!' said I. 'Ah,' said he, 'it's very neat—very neat. And it isn't an answer that would occur to everybody. Very neat indeed.' A few yards further on, I fell in with Smith and to him I propounded the same riddle. He frowned over it for a minute, and then gave it up. Meekly

I faltered out the answer. 'A poor thing, sir!' Smith growled, as he turned away. 'A very poor thing! I wonder you care to repeat such rubbish!' Yet Smith's mind is, if possible, even more colossal than Brown's.

The second Problem of Knot VI. is an example in ordinary Double Rule of Three, whose essential feature is that the result depends on the variation of several elements, which are so related to it that, if all but one be constant, it varies as that one: hence, if none be constant, it varies as their product. Thus, for example, the cubical contents of a rectangular tank vary as its length, if breadth and depth be constant, and so on; hence, if none be constant, it varies as the product of the length, breadth, and depth.

When the result is not thus connected with the varying elements, the Problem ceases to be Double Rule of Three and often becomes one of great complexity.

To illustrate this, let us take two candidates for a prize, *A* and *B*, who are to compete in French, German, and Italian:

(*a*) Let it be laid down that the result is to depend on their *relative* knowledge of each subject, so that, whether their marks, for French, be '1, 2' or '100, 200,' the result will be the same: and let it also be laid down that, if they get equal marks on 2 papers, the final marks are to have the same ratio as those of the 3rd paper. This is a case of ordinary Double Rule of Three. We multiply *A*'s 3 marks together, and do the same for *B*. Note that, if *A* gets a single '0,' his final mark is '0,' even if he gets full marks for 2 papers while *B* gets only one mark for each paper. This of course would be very unfair on *A*, though a correct solution under the given conditions.

(*b*) The result is to depend, as before, on *relative* knowledge; but French is to have twice as much weight as German or Italian. This is an unusual form of question. I should be inclined to say 'the resulting ratio is to be nearer to the French ratio than if we multiplied as in (*a*), and so much nearer that it would be necessary to use the other multipliers *twice* to produce the same result as in (*a*):' *e.g.* if the French Ratio were $\frac{2}{10}$, and the others $\frac{2}{9}$, $\frac{1}{9}$ so that the ultimate ratio, by method (*a*), would be $\frac{2}{45}$, I should multiply instead by $\frac{2}{3}$, $\frac{1}{3}$, giving the result, $\frac{1}{3}$ which is nearer to $\frac{2}{10}$ than if he had used method (*a*).

(*c*) The result is to depend on *actual* amount of knowledge of the 3 subjects collectively. Here we have to ask two questions. (1) What is to be the 'unit' (*i.e.* 'standard to measure by') in each subject? (2) Are these units to be of equal, or unequal value? The usual 'unit' is the knowledge shown by answering the whole paper correctly; calling this '100,' all lower amounts are represented by numbers between '0' and '100.' Then, if these units are to be of equal value, we simply add *A*'s 3 marks together, and do the same for *B*.

(*d*) The conditions are the same as (*c*), but French is to have double weight. Here we simply double the French marks, and add as before.

(*e*) French is to have such weight, that, if other marks be equal, the ultimate ratio is to be that of the French paper, so that a '0' in this would swamp the candidate: but the other two subjects are only to affect the result collectively, by the amount of knowledge shown, the two being reckoned of equal value. Here I should add *A*'s German and Italian marks together, and multiply by his French mark.

But I need not go on: the problem may evidently be set with many varying conditions, each requiring its own method of solution. The Problem in Knot VI. was meant to belong to variety (*a*), and to make this clear, I inserted the following passage:

'Usually the competitors differ in one point only. Thus, last year, Fifi and Gogo made the same number of scarves in the trial week, and they were equally light; but Fifi's were twice as warm as Gogo's, and she was pronounced twice as good.'

What I have said will suffice, I hope, as an answer to Balbus, who holds that (*a*) and (*c*) are the only possible varieties of the problem, and that to say 'We cannot use addition, therefore we must be intended to use multiplication,' is 'no more illogical than, from knowledge that one was not born in the night, to infer that he was born in the daytime'; and also to Fifee, who says 'I think a little more consideration will show you that our "error of *adding* the proportional numbers together for each candidate instead of *multiplying*" is no error at all.' Why, even if addition *had* been the right method to use, not one of the writers (I speak from memory) showed any consciousness of the necessity of fixing a 'unit' for each subject. 'No error at all!' They were positively steeped in error!

One correspondent (I do not name him, as the communication is not quite friendly in tone) writes thus:—'I wish to add, very respectfully, that I think it would be in better taste if you were to abstain from the very trenchant expressions which you are accustomed to indulge in when criticising the answer. That such a tone must not be' ('be not'?) 'agreeable to the persons concerned who have made mistakes may possibly have no great weight with you, but I hope you will feel that it would be as well not to employ it, *unless you are quite certain of being correct* *yourself.*' The only instances the writer gives of the 'trenchant expressions' are 'hapless' and 'malefactors.' I beg to assure him (and any others who may need the assurance: I trust there are none) that all such words have been used in jest, and with no idea that they could possibly annoy any one, and that I sincerely regret any annoyance I may have thus inadvertently given. May I hope that in future they will recognise the distinction between severe language used in sober earnest, and the 'words of unmeant bitterness,' which Coleridge has alluded to in that lovely passage beginning 'A little child, a limber elf'? If the writer will refer to that passage, or to the preface to 'Fire, Famine, and Slaughter,' he will find the distinction, for which I plead, far better drawn out than I could hope to do in any words of mine.

The writer's insinuation that I care not how much annoyance I give to my readers I think it best to pass over in silence; but to his concluding remark I must entirely demur. I hold that to use language likely to annoy any of my correspondents would not be in the least justified by the plea that I was 'quite certain of being correct.' I trust that the knot-untiers and I are not on such terms as those!

I beg to thank *G. B.* for the offer of a puzzle—which, however, is too like the old one 'Make four 9's into 100.'

ANSWERS TO KNOT VIII.

§ 1. The Pigs.

Problem.—Place twenty-four pigs in four sties so that, as you go round and round, you may always find the number in each sty nearer to ten than the number in the last.

Answer.—Place 8 pigs in the first sty, 10 in the second, nothing in the third, and 6 in the fourth: 10 is nearer ten than 8; nothing is nearer ten than 10; 6 is nearer ten than nothing; and 8 is nearer ten than 6.

This problem is noticed by only two correspondents. Balbus says 'it certainly cannot be solved mathematically, nor do I see how to solve it by any verbal quibble.' Nolens Volens makes Her Radiancy change the direction of going round; and even then is obliged to add 'the pigs must be carried in front of her'!

§ 2. The Grurmstipths.

Problem.—Omnibuses start from a certain point, both ways, every 15 minutes. A traveller, starting on foot along with one of them, meets one in 12½ minutes: when will he be overtaken by one?

Answer.—In 6¼ minutes.

☼

Solution.—Let '*a*' be the distance an omnibus goes in 15 minutes, and '*x*' the distance from the starting-point to where the traveller is overtaken. Since the omnibus met is due at the starting-point in 2½ minutes, it goes in that time as far as the traveller walks in 12½; *i.e.* it goes 5 times as fast. Now the overtaking omnibus is '*a*' behind the traveller when he starts, and therefore goes '$a + x$' while he goes 'x.' Hence $a + x = 5x$; *i.e.* $4x = a$, and $x = a/4$. This distance would be traversed by an omnibus in 15⁄4 minutes, and therefore by the traveller in 5 × 15⁄4. Hence he is overtaken in 18¾ minutes after starting, *i.e.* in 6¼ minutes after meeting the omnibus.

☼

Four answers have been received, of which two are wrong. Dinah Mite rightly states that the overtaking omnibus reached the point where they met the other omnibus 5 minutes after they left, but wrongly concludes that, going 5 times as fast, it would overtake them in another minute. The travellers are 5-minutes-walk ahead of the omnibus, and must walk ¼ of this distance farther before the omnibus overtakes them, which will be ⅕ of the distance traversed by the omnibus in the same time: this will require 1¼ minutes more. Nolens Volens tries it by a process like 'Achilles and the Tortoise.' He rightly states that, when the overtaking omnibus leaves the gate, the travellers are ⅕ of '*a*' ahead, and that it will take the omnibus 3 minutes to traverse this distance; 'during which time' the travellers, he tells us, go 1⁄15 of '*a*' (this should be 1⁄25). The travellers being now 1⁄15 of '*a*' ahead, he concludes that the work remaining to be done is for the travellers to go 1⁄60 of '*a*,' while the omnibus goes 1⁄12. The *principle* is correct, and might have been applied earlier.

CLASS LIST.

I.

Balbus.

Delta.

ANSWERS TO KNOT IX.

§ 1. The Buckets.

Problem.—Lardner states that a solid, immersed in a fluid, displaces an amount equal to itself in bulk. How can this be true of a small bucket floating in a larger one?

Solution.—Lardner means, by 'displaces,' 'occupies a space which might be filled with water without any change in the surroundings.' If the portion of the floating bucket, which is above the water, could be annihilated, and the rest of it transformed into water, the surrounding water would not change its position: which agrees with Lardner's statement.

☼

Five answers have been received, none of which explains the difficulty arising from the well-known fact that a floating body is the same weight as the displaced fluid. Hecla says that 'only that portion of the smaller bucket which descends below the original level of the water can be properly said to be immersed, and only an equal bulk of water is displaced.' Hence, according to Hecla, a solid, whose weight was equal to that of an equal bulk of water, would not float till the whole of it was below 'the original level' of the water: but, as a matter of fact, it would float as soon as it was all under water. Magpie says the fallacy is 'the assumption that one body can displace another from a place where it isn't,' and that Lardner's assertion is incorrect, except when the containing vessel 'was originally full to the brim.' But the question of floating depends on the present state of things, not on past history. OLD KING COLE takes the same view as HECLA. TYMPANUM and VINDEX assume that 'displaced' means 'raised above its original level,' and merely explain how it comes to pass that the water, so raised, is less in bulk than the immersed portion of bucket, and thus land themselves—or rather set themselves floating—in the same boat as HECLA.

I regret that there is no Class-list to publish for this Problem.

§ 2. BALBUS'S ESSAY.

Problem.—Balbus states that if a certain solid be immersed in a certain vessel of water, the water will rise through a series of distances, two inches, one inch, half an inch, etc., which series has no end. He concludes that the water will rise without limit. Is this true?

Solution.—No. This series can never reach 4 inches, since, however many terms we take, we are always short of 4 inches by an amount equal to the last term taken.

Three answers have been received—but only two seem to me worthy of honours.

Tympanum says that the statement about the stick 'is merely a blind, to which the old answer may well be applied, *solvitur ambulando*, or rather *mergendo*.' I trust Tympanum will not test this in his own person, by taking the place of the man in Balbus's Essay! He would infallibly be drowned.

Old King Cole rightly points out that the series, 2, 1, etc., is a decreasing Geometrical Progression: while Vindex rightly identifies the fallacy as that of 'Achilles and the Tortoise.'

CLASS LIST.

I.

Old King Cole.

Vindex.

§ 3. The Garden.

Problem.—An oblong garden, half a yard longer than wide, consists entirely of a gravel-walk, spirally arranged, a yard wide and 3,630 yards long. Find the dimensions of the garden.

Answer.—60, 60½.

Solution.—The number of yards and fractions of a yard traversed in walking along a straight piece of walk, is evidently the same as the number of square-yards and fractions of a square-yard, contained in that piece of walk: and the distance, traversed in passing through a square-yard at a corner, is evidently a yard. Hence the area of the garden is 3,630 square-yards: *i.e.*, if x be the width, $x(x + ½) = 3{,}630$. Solving this Quadratic, we find $x = 60$. Hence the dimensions are 60, 60½.

Twelve answers have been received—seven right and five wrong.

C. G. L., Nabob, Old Crow, and Tympanum assume that the number of yards in the length of the path is equal to the number of square-yards in the garden. This is true, but should have been proved. But each is guilty of darker deeds. C. G. L.'s 'working' consists of dividing 3,630 by 60. Whence came this divisor, oh Segiel? Divination? Or was it a dream? I fear this solution is worth nothing. Old Crow's is shorter, and so (if possible) worth rather less. He says the answer 'is at once seen to be 60 × 60½'! Nabob's calculation is short, but 'as rich as a Nabob' in error. He says that the square root of 3,630, multiplied by 2, equals the length plus the breadth. That is 60.25 × 2 = 120½. His first assertion is only true of a *square* garden. His second is irrelevant, since 60.25 is *not* the square-root of 3,630! Nay, Bob, this will *not* do! Tympanum says that, by extracting the square-root of 3,630, we get 60 yards with a remainder of 30/60, or half-a-yard, which we add so as to make the oblong 60 × 60½. This is very terrible: but worse remains behind. Tympanum proceeds thus:—'But why should there be the half-yard at all? Because without it there would be no space at all for flowers. By means of it, we find reserved in the very centre a small plot of ground, two yards long by half-a-yard wide, the only space not occupied by walk.' But Balbus expressly said that the walk 'used up the whole of the area.' Oh, Tympanum! My tympa is exhausted: my brain is num! I can say no more.

Hecla indulges, again and again, in that most fatal of all habits in computation—the making *two* mistakes which cancel each other. She takes x as the width of the garden, in yards, and $x + ½$ as its length, and makes her first 'coil' the sum of $x½$, $x½$, x-1, x-1, *i.e.* 4x-3: but the fourth term should be x-1½, so that her first coil is ½ a yard too long. Her second coil is the sum of x-2½, x-2½, x-3, x-3: here the first term should be x-2 and the last x-3½: these two mistakes cancel, and this coil is therefore right. And the same thing is true of every other coil but the last, which needs an extra half-yard to reach the *end* of the path: and this exactly balances the mistake in the first coil. Thus the sum total of the coils comes right though the working is all wrong.

Of the seven who are right, Dinah Mite, Janet, Magpie, and Taffy make the same assumption as C. G. L. and Co. They then solve by a Quadratic. Magpie also tries it by Arithmetical Progression, but fails to notice that the first and last 'coils' have special values.

Alumnus Etonæ attempts to prove what C. G. L. assumes by a particular instance, taking a garden 6 by 5½. He ought to have proved it generally: what is true of one number is not always true of others. Old King Cole solves it by an Arithmetical Progression. It is right, but too lengthy to be worth as much as a Quadratic.

Vindex proves it very neatly, by pointing out that a yard of walk measured along the middle represents a square yard of garden, 'whether we consider the straight stretches of walk or the square yards at the angles, in which the middle line goes half a yard in one direction and then turns a right angle and goes half a yard in another direction.'

CLASS LIST.

I.

Vindex.

II.

Alumnus Etonæ.

Old King Cole.

III.

Dinah Mite.

Janet.

Magpie.

Taffy.

ANSWERS TO KNOT X.

§ 1. The Chelsea Pensioners.

Problem.—If 70 per cent have lost an eye, 75 per cent an ear, 80 per cent an arm, 85 per cent a leg: what percentage, *at least*, must have lost all four?

Answer.—Ten.

Solution.—(I adopt that of Polar Star, as being better than my own). Adding the wounds together, we get 70 + 75 + 80 + 85 = 310, among 100 men; which gives 3 to each, and 4 to 10 men. Therefore the least percentage is 10.

☼

Nineteen answers have been received. One is '5,' but, as no working is given with it, it must, in accordance with the rule, remain 'a deed without a name.' Janet makes it '35 and 7⁄10ths.' I am sorry she has misunderstood the question, and has supposed that those who had lost an ear were 75 per cent *of those who had lost an eye*; and so on. Of course, on this supposition, the percentages must all be multiplied together. This she has done correctly, but I can give her no honours, as I do not think the question will fairly bear her interpretation, Three Score and Ten makes it '19 and 3⁄8ths.' Her solution has given me—I will not say 'many anxious days and sleepless nights,' for I wish to be strictly truthful, but—some trouble in making any sense at all of it. She makes the number of 'pensioners wounded once' to be 310 ('per cent,' I suppose!): dividing by 4, she gets 77 and a half as 'average percentage:' again dividing by 4, she gets 19 and ⅜ths as 'percentage wounded four times.' Does she suppose wounds of different kinds to 'absorb' each other, so to speak? Then, no doubt, the *data* are equivalent to 77 pensioners with one wound each, and a half-pensioner with a half-wound. And does she then suppose these concentrated wounds to be *transferable*, so that ¾ths of these unfortunates can obtain perfect health by handing over their wounds to the remaining ¼th? Granting these suppositions, her answer is right; or rather, *if* the question had been 'A road is covered with one inch of gravel, along 77 and a half per cent of it. How much of it could be covered 4 inches deep with the same material?' her answer *would* have been right. But alas, that *wasn't* the question! Delta makes some most amazing assumptions: 'let every one who has not lost an eye have lost an ear,' 'let every one who has not lost both eyes and ears have lost an arm.' Her ideas of a battle-field are grim indeed. Fancy a warrior who would continue fighting after losing both eyes, both ears, and both arms! This is a case which she (or 'it?') evidently considers *possible*.

Next come eight writers who have made the unwarrantable assumption that, because 70 per cent have lost an eye, *therefore* 30 per cent have *not* lost one, so that they have *both* eyes. This is illogical. If you give me a bag containing 100 sovereigns, and if in an hour I come to you (my face *not* beaming with gratitude nearly so much as when I received the bag) to say 'I am sorry to tell you that 70 of these sovereigns are bad,' do I thereby guarantee the other 30 to be good? Perhaps I have not tested them yet. The sides of this illogical octagon are as follows, in alphabetical order:—Algernon Bray, Dinah Mite, G. S. C., Jane E., J. D. W., Magpie (who makes the delightful remark 'therefore 90 per cent have two of something,' recalling to one's memory that fortunate monarch, with whom Xerxes was so much pleased that 'he gave him ten of everything!'), S. S. G., and Tokio.

Bradshaw of the Future and T. R. do the question in a piecemeal fashion—on the principle that the 70 per cent and the 75 per cent, though commenced at opposite ends of the 100, must overlap by *at least* 45 per cent; and so on. This is quite correct working, but not, I think, quite the best way of doing it.

The other five competitors will, I hope, feel themselves sufficiently glorified by being placed in the first class, without my composing a Triumphal Ode for each!

CLASS LIST.

I.

Old Cat.

Old Hen.

Polar Star.

Simple Susan.

White Sugar.

II.

Bradshaw of the Future.

T. R.

III.

Algernon Bray.

Dinah Mite.

G. S. C.

Jane E.

J. D. W.

Magpie.

S. S. G.

Tokio.

§ 2. Change of Day.

I must postpone, *sine die*, the geographical problem—partly because I have not yet received the statistics I am hoping for, and partly because I am myself so entirely puzzled by it; and when an examiner is himself dimly hovering between a second class and a third how is he to decide the position of others?

§ 3. The Sons' Ages.

Problem.—At first, two of the ages are together equal to the third. A few years afterwards, two of them are together double of the third. When the number of years since the first occasion is two-thirds of the sum of the ages on that occasion, one age is 21. What are the other two?

Answer.—15 and 18.

☼

Solution.—Let the ages at first be x, y, $(x + y)$. Now, if $a + b = 2c$, then $(a-n) + (b-n) = 2(c-n)$, whatever be the value of n. Hence the second relationship, if *ever* true, was *always* true. Hence it was true at first. But it cannot be true that x and y are together double of $(x + y)$. Hence it must be true of $(x + y)$, together with x or y; and it does not matter which we take. We assume, then, $(x + y) + x = 2y$; *i.e.* $y = 2x$. Hence the three ages were, at first, x, $2x$, $3x$; and the number of years, since that time is two-thirds of $6x$, *i.e.* is $4x$. Hence the present ages are $5x$, $6x$, $7x$. The ages are clearly *integers*, since this is only 'the year when one of my sons comes of age.' Hence $7x = 21$, $x = 3$, and the other ages are 15, 18.

Eighteen answers have been received. One of the writers merely asserts that the first occasion was 12 years ago, that the ages were then 9, 6, and 3; and that on the second occasion they were 14, 11, and 8! As a Roman father, I *ought* to withhold the name of the rash writer; but respect for age makes me break the rule: it is THREE SCORE AND TEN. JANE E. also asserts that the ages at first were 9, 6, 3: then she calculates the present ages, leaving the *second* occasion unnoticed. OLD HEN is nearly as bad; she 'tried various numbers till I found one that fitted *all* the conditions'; but merely scratching up the earth, and pecking about, is *not* the way to solve a problem, oh venerable bird! And close after OLD HEN prowls, with hungry eyes, OLD CAT, who calmly assumes, to begin with, that the son who comes of age is the *eldest*. Eat your bird, Puss, for you will get nothing from me!

There are yet two zeroes to dispose of. MINERVA assumes that, on *every* occasion, a son comes of age; and that it is only such a son who is 'tipped with gold.' Is it wise thus to interpret 'now, my boys, calculate your ages, and you shall have the money'? Bradshaw of the Future says 'let' the ages at first be 9, 6, 3, then assumes that the second occasion was 6 years afterwards, and on these baseless assumptions brings out the right answers. Guide *future* travellers, an thou wilt: thou art no Bradshaw for *this* Age!

Of those who win honours, the merely 'honourable' are two. Dinah Mite ascertains (rightly) the relationship between the three ages at first, but then *assumes* one of them to be '6,' thus making the rest of her solution tentative. M. F. C. does the algebra all right up to the conclusion that the present ages are $5z$, $6z$, and $7z$; it then assumes, without giving any reason, that $7z = 21$.

Of the more honourable, Delta attempts a novelty—to discover *which* son comes of age by elimination: it assumes, successively, that it is the middle one, and that it is the youngest; and in each case it *apparently* brings out an absurdity. Still, as the proof contains the following bit of algebra, '$63 = 7x + 4y$; $21 = x + 4$ sevenths of y,' I trust it will admit that its proof is not *quite* conclusive. The rest of its work is good. Magpie betrays the deplorable tendency of her tribe—to appropriate any stray conclusion she comes across, without having any *strict* logical right to it. Assuming A, B, C, as the ages at first, and D as the number of the years that have elapsed since then, she finds (rightly) the 3 equations, $2A = B$, $C = B + A$, $D = 2B$. She then says 'supposing that $A = 1$, then $B = 2$, $C = 3$, and $D = 4$. Therefore for A, B, C, D, four numbers are wanted which shall be to each other as 1:2:3:4.' It is in the 'therefore' that I detect the unconscientiousness of this bird. The conclusion *is* true, but this is only because the equations are 'homogeneous' (*i.e.* having one 'unknown' in each term), a fact which I strongly suspect had not been grasped—I beg pardon, clawed—by her. Were I to lay this little pitfall, '$A + 1$

= B, $B + 1 = C$; supposing $A = 1$, then $B = 2$ and $C = 3$. *Therefore* for A, B, C, three numbers are wanted which shall be to one another as 1:2:3,' would you not flutter down into it, oh Magpie, as amiably as a Dove? Simple Susan is anything but simple to *me*. After ascertaining that the 3 ages at first are as 3:2:1, she says 'then, as two-thirds of their sum, added to one of them, = 21, the sum cannot exceed 30, and consequently the highest cannot exceed 15.' I suppose her (mental) argument is something like this:—'two-thirds of sum, + one age, = 21; ∴ sum, + 3 halves of one age, = 31 and a half. But 3 halves of one age cannot be less than 1 and-a-half (here I perceive that Simple Susan would on no account present a guinea to a new-born baby!) hence the sum cannot exceed 30.' This is ingenious, but her proof, after that, is (as she candidly admits) 'clumsy and roundabout.' She finds that there are 5 possible sets of ages, and eliminates four of them. Suppose that, instead of 5, there had been 5 million possible sets? Would Simple Susan have courageously ordered in the necessary gallon of ink and ream of paper?

The solution sent in by C. R. is, like that of Simple Susan, partly tentative, and so does not rise higher than being Clumsily Right.

Among those who have earned the highest honours, Algernon Bray solves the problem quite correctly, but adds that there is nothing to exclude the supposition that all the ages were *fractional*. This would make the number of answers infinite. Let me meekly protest that I *never* intended my readers to devote the rest of their lives to writing out answers! E. M. Rix points out that, if fractional ages be admissible, any one of the three sons might be the one 'come of age'; but she rightly rejects this supposition on the ground that it would make the problem indeterminate. White Sugar is the only one who has detected an oversight of mine: I had forgotten the possibility (which of course ought to be allowed for) that the son, who came of age that *year*, need not have done so by that *day*, so that he *might* be only 20. This gives a second solution, viz., 20, 24, 28. Well said, pure Crystal! Verily, thy 'fair discourse hath been as sugar'!

CLASS LIST.

I.

Algernon Bray.

An Old Fogey.

E. M. Rix.

G. S. C.

S. S. G.

Tokio.

T. R.

White Sugar.

II.

C. R.

Delta.

Magpie.

Simple Susan.

III.

Dinah Mite.

M. F. C.

I have received more than one remonstrance on my assertion, in the Chelsea Pensioners' problem, that it was illogical to assume, from the *datum* '70 p. c. have lost an eye,' that 30 p. c. have *not*. Algernon Bray states, as a parallel case, 'suppose Tommy's father gives him 4 apples, and he eats one of them, how many has he left?' and says 'I think we are justified in answering, 3.' I think so too. There is no 'must' here, and the *data* are evidently meant to fix the answer *exactly*: but, if the question were set me 'how many *must* he have left?', I should understand the *data* to be that his father gave him 4 *at least*, but *may* have given him more.

I take this opportunity of thanking those who have sent, along with their answers to the Tenth Knot, regrets that there are no more Knots to come, or petitions that I should recall my resolution to bring them to an end. I am most grateful for their kind words; but I think it wisest to end what, at best, was but a lame attempt. 'The stretched metre of an antique song' is beyond my compass; and my puppets were neither distinctly *in* my life (like those I now address), nor yet (like Alice and the Mock Turtle) distinctly *out* of it. Yet let me at least fancy, as I lay down the pen, that I carry with me into my silent life, dear reader, a farewell smile from your unseen face, and a kindly farewell pressure from your unfelt hand! And so, good night! Parting is such sweet sorrow, that I shall say 'good night!' till it be morrow.